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ON THE ASYMPTOTIC SOLUTION OF MIXED THREE-DIMENSIONAL PROBLEMS FOR DOUBLE-LAYER ANISOTROPIC PLATES*

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An asymptotic method /1/ is used to solve mixed three-dimensional problems for double-layer anisotropic thermoelastic plates, and the possibility of using the results obtained in computations of elastic bases and foundations in the model of a compressible layer is examined. Additional hypotheses on the nature of the displacement field distribution are not required here, as was done earlier /2/ in the analysis of plates and shells on an elastic basis in the model of a compressible layer. Exact solutions of the internal problem are obtained in a number of cases. This approach can be extended to the case of multilayer plates and shells.

1. The following problem is posed: it is required to find the solution of the equations of the spatial problem of the theory of elasticity of an anisotropic layer in a domain occupied by a double-layer plate $\Omega = \{\alpha, \beta, \gamma: \alpha, \beta \in \Omega_0, -h_2 \leqslant \gamma \leqslant h_1\}$, where Ω_0 is the separation plane, and h_1, h_2 are the layer thicknesses (Fig.1). Specified volume forces $F_\alpha^{(i)}(\alpha, \beta, \gamma)$ (α, β, γ) and temperature effects, whose influence is taken into account by the theory of temperature stresses using the Duhamel-Neumann law /3/, act on the plate. The quantities referring to the upper layer are denoted by the superscript (1), and to the lower layer by the superscript (2). On the facial surface of the lower layer $\gamma = -h_2$ values of the displacements are given, in particular, it is rigidly clamped (the displacement vector on the surface equals zero)

$$u_\alpha^{(2)}(-h_2) = u^-(\alpha, \beta), \quad u_\beta^{(2)}(-h_2) = v^-(\alpha, \beta), \quad u_\gamma^{(2)}(-h_2) = w^+(\alpha, \beta) \tag{1.1}$$

while on the plane $\gamma = h_1$ one of the combinations of the following conditions is given

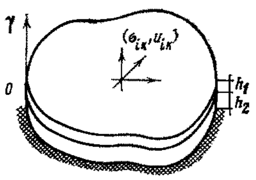

Fig.1

$$u_\alpha^{(1)}(h_1) = u^+(\alpha, \beta), \quad u_\beta^{(1)}(h_1) = v^+(\alpha, \beta), \quad u_\gamma^{(1)}(h_1) = w^+(\alpha, \beta) \tag{1.2}$$

$$\sigma_{\gamma\gamma}^{(1)}(h_1) = \varepsilon^{-1}\sigma_{\gamma\gamma}^+(\alpha, \beta), \quad \sigma_{\beta\gamma}^{(1)}(h_1) = \varepsilon^{-1}\sigma_{\beta\gamma}^+(\alpha, \beta), \quad \sigma_{\alpha\gamma}^{(1)}(h_1) = \varepsilon^{-1}\sigma_{\alpha\gamma}^+(\alpha, \beta) \tag{1.3}$$

$$\sigma_{\alpha\gamma}^{(1)}(h_1) = \varepsilon^{-1}\sigma_{\alpha\gamma}^+(\alpha, \beta), \quad \sigma_{\beta\gamma}^{(1)}(h_1) = \varepsilon^{-1}\sigma_{\beta\gamma}^+(\alpha, \beta), \quad u_\gamma(h_1) = w^+(\alpha, \beta) \tag{1.4}$$

$$u_\alpha^{(1)}(h_1) = u^+(\alpha, \beta), \quad u_\beta^{(1)}(h_1) = v^+(\alpha, \beta), \quad \sigma_{\gamma\gamma}^{(1)}(h_1) = \varepsilon^{-1}\sigma_{\gamma\gamma}^+(\alpha, \beta) \tag{1.5}$$

The anisotropy is common and is characterised by 21 elastic constants for each layer. The solution found should also satisfy conditions of the lateral surface S_Ω which for the moment are considered to be arbitrary. It will be seen from the subsequent exposition that these conditions do not influence the internal state of stress in the problems under consideration; they are due to the boundary layer.

In particular, problem (1.1), (1.3) models the elastic basis-foundation in the model of a compressible layer /2, 4/.

We introduce the dimensionless variables $\xi = \alpha/a$, $\eta = \beta/a$, $\zeta = \gamma/h_2 = \varepsilon^{-1}\gamma/a$ $(\varepsilon = h_2/a,\ h_1 + h_2 \ll a$; (if $h_1 > h_2$ it is best to introduce $\zeta = \gamma/h_1$, $\varepsilon = h_1/a$) and the dimensionless displacements $u^{(i)} = u_\alpha^{(i)}/a$, $v^{(i)} = u_\beta^{(i)}/a$, $w^{(i)} = u_\gamma^{(i)}/a$, where a is the characteristic dimension of the plate, $i = 1$ for the upper layer and $i = 2$ for the lower. The system of equations of the spatial problem /3, 5/ (the equilibrium equations, the elasticity relations and the displacement deformation) takes the form

$$\frac{1}{A}\frac{\partial\sigma_{\alpha\alpha}^{(i)}}{\partial\xi} + \frac{1}{B}\frac{\partial\sigma_{\alpha\beta}^{(i)}}{\partial\eta} + \varepsilon^{-1}\frac{\partial\sigma_{\alpha\gamma}^{(i)}}{\partial\zeta} + a(\sigma_{\alpha\alpha}^{(i)} - \sigma_{\beta\beta}^{(i)})k_\beta + \tag{1.6}$$
$$2ak_\alpha\sigma_{\alpha\beta}^{(i)} + aF_\alpha^{(i)} = 0 \quad (\alpha, \beta; \xi, \eta)$$
$$\frac{1}{A}\frac{\partial\sigma_{\alpha\gamma}^{(i)}}{\partial\xi} + \frac{1}{B}\frac{\partial\sigma_{\beta\gamma}^{(i)}}{\partial\eta} + \varepsilon^{-1}\frac{\partial\sigma_{\gamma\gamma}^{(i)}}{\partial\zeta} + ak_\beta\sigma_{\alpha\gamma}^{(i)} + ak_\alpha\sigma_{\beta\gamma}^{(i)} + aF_\gamma^{(i)} = 0$$
$$\frac{1}{A}\frac{\partial u^{(i)}}{\partial\xi} + ak_\alpha v^{(i)} = a_{11}^{(i)}\sigma_{\alpha\alpha}^{(i)} + a_{12}^{(i)}\sigma_{\beta\beta}^{(i)} + a_{13}^{(i)}\sigma_{\gamma\gamma}^{(i)} + \ldots + a_{16}^{(i)}\sigma_{\alpha\beta}^{(i)} + \alpha_{11}^{(i)}\theta^{(i)} \quad (1, 2; \alpha, \beta; u, v)$$

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$$\frac{1}{A}\frac{\partial w^{(i)}}{\partial \xi}+\varepsilon^{-1}\frac{\partial u^{(i)}}{\partial \zeta}=a_{15}^{(i)}\sigma_{\alpha\alpha}^{(i)}+a_{25}^{(i)}\sigma_{\beta\beta}^{(i)}+\ldots+a_{56}^{(i)}\sigma_{\alpha\beta}^{(i)}+\alpha_{13}^{(i)}\theta^{(i)}$$

$$\varepsilon^{-1}\frac{\partial w^{(i)}}{\partial \zeta}=a_{13}^{(i)}\sigma_{\alpha\alpha}^{(i)}+a_{23}^{(i)}\sigma_{\beta\beta}^{(i)}+\ldots+a_{36}^{(i)}\sigma_{\alpha\beta}^{(i)}+\alpha_{33}^{(i)}\theta^{(i)}$$

$$\frac{1}{B}\frac{\partial u^{(i)}}{\partial \eta}+\frac{1}{A}\frac{\partial v^{(i)}}{\partial \xi}-a\left(k_\alpha u^{(i)}+k_\beta v^{(i)}\right)=a_{16}^{(i)}\sigma_{\alpha\alpha}^{(i)}+\ldots+a_{66}^{(i)}\sigma_{\alpha\beta}^{(i)}+\alpha_{12}^{(i)}\theta^{(i)}$$

$$\frac{1}{B}\frac{\partial w^{(i)}}{\partial \eta}+\varepsilon^{-1}\frac{\partial v^{(i)}}{\partial \zeta}=a_{14}^{(i)}\sigma_{\alpha\alpha}^{(i)}+a_{24}^{(i)}\sigma_{\beta\beta}^{(i)}+\ldots+a_{46}^{(i)}\sigma_{\alpha\beta}^{(i)}+\alpha_{23}^{(i)}\theta^{(i)}$$

Here A, B are coefficients of the first quadratic form, k_α, k_β are curvatures, a_{jk} are elastic compliance coefficients, α_{jk} are the coefficients of thermal expansion, and $\theta^{(i)}=T^{(i)}-T_0^{(i)}$ is the temperature increment. Variability over the coordinate lines of the middle surface can also be taken into account in (1.6) in a known manner. System (1.6) is perturbed singularly by the small parameter ε. Its solution is comprised of the solution of the internal problem and the boundary layer /1, 6/.

We will seek the solution of the internal problem, or the penetrating solution in the form

$$Q^{(i)}=\Sigma\varepsilon^{\varkappa_Q+s}Q^{(i),s}(\xi,\eta,\zeta),\quad i=1,2 \tag{1.7}$$

where $Q^{(i)}$ is any of the quantities desired, and the summation is over s between zero and the number of the approximation N. To determine the unknown coefficients $Q^{(i),s}$ we obtain a non-contradictory system after substituting (1.7) into (1.6), if /7/

$$\varkappa_Q=-1 \text{ for the stresses} \quad \varkappa_Q=0 \text{ for the displacements} \tag{1.8}$$

In principle, the asymptotic form of the stresses and displacements corresponding to conditions (1.8) differs from the asymptotic form of the same quantities in the classical theory of both isotropic and anisotropic plates /1, 8/, i.e., when values of the stress are given on the facial surfaces. In the problems under consideration here, the stresses generally turn out to be equally correct. Let

$$F_\alpha^{(i)}=\Sigma\varepsilon^{-2+s}F_{\alpha s}^{(i)}(\xi,\eta,\zeta)\ (\alpha,\beta,\gamma),\quad \theta^{(i)}=\Sigma\varepsilon^{-1+s}\theta_s^{(i)}(\xi,\eta,\zeta) \tag{1.9}$$

Substituting (1.7) into (1.6) and taking account of (1.8) and (1.9), we obtain the system of governing equations in $Q^{(i),s}$ by the usual procedure. This system can be integrated with respect to the variable ζ. Then by satisfying conditions (1.1) and the elastic contact conditions for $\gamma=0$

$$u^{(1)}=u^{(2)},\quad v^{(1)}=v^{(2)},\quad w^{(1)}=w^{(2)},\quad \sigma_{\alpha\gamma}^{(1)}=\sigma_{\alpha\gamma}^{(2)},\ (\alpha,\beta,\gamma)\quad \text{when}\quad \gamma=0 \tag{1.10}$$

we obtain for $Q^{(i),s}$

$$\begin{aligned}
&\sigma_{\alpha\alpha}^{(i),s}=A_{13}^{(i)}\sigma_{\gamma\gamma0}^{(s)}+A_{14}^{(i)}\sigma_{\beta\gamma0}^{(s)}+A_{15}^{(i)}\sigma_{\alpha\gamma0}^{(s)}+\sigma_{\alpha\alpha*}^{(i),s}(\xi,\eta,\zeta)\qquad(\alpha,\beta;1,2)\\
&\sigma_{\alpha\beta}^{(i),s}=A_{63}^{(i)}\sigma_{\gamma\gamma0}^{(s)}+A_{64}^{(i)}\sigma_{\beta\gamma0}^{(s)}+A_{65}^{(i)}\sigma_{\alpha\gamma0}^{(s)}+\sigma_{\alpha\beta*}^{(i),s}(\xi,\eta,\zeta)\\
&\sigma_{\alpha\gamma}^{(i),s}=\sigma_{\alpha\gamma0}^{(s)}+\sigma_{\alpha\gamma*}^{(i)}(\xi,\eta,\zeta)\ \ (\alpha,\beta),\quad \sigma_{\gamma\gamma}^{(i),s}=\sigma_{\gamma\gamma0}^{(s)}+\sigma_{\gamma\gamma*}^{(i),s}(\xi,\eta,\zeta)\\
&u^{(i),s}=D_{53}^{(i)}\sigma_{\gamma\gamma0}^{(s)}+D_{54}^{(i)}\sigma_{\beta\gamma0}^{(s)}+D_{55}^{(i)}\sigma_{\alpha\gamma0}^{(s)}+u^{-(s)}-u_*^{(2),s}(\zeta=-1)+\\
&\quad u_*^{(i),s}(\xi,\eta,\zeta)\\
&(u,v,w;\ 53,43,33;\ 54,44,34;\ 55,45,35),\ i=1,2
\end{aligned} \tag{1.11}$$

Here

$$\begin{aligned}
&u^{\pm(0)}=u^{\pm}(a\xi,a\eta)/a,\quad u^{\pm(s)}=0,\quad s>0\quad(u,v,w)\\
&A_{1j}^{(i)}=[a_{1j}^{(i)}B_{26}^{(i)}+a_{2j}^{(i)}B_{12}^{(i)}+a_{6j}^{(i)}B_{62}^{(i)}]/\Delta^{(i)},\quad A_{2j}^{(i)}=[a_{1j}^{(i)}B_{12}^{(i)}+a_{2j}^{(i)}B_{16}^{(i)}+a_{6j}^{(j)}B_{61}^{(i)}]/\Delta^{(i)}\\
&A_{6j}^{(i)}=[a_{1j}^{(i)}B_{62}^{(i)}+a_{2j}^{(i)}B_{61}^{(i)}+a_{6j}^{(i)}A_{12}^{(i)}]/\Delta^{(i)},\quad A_{12}^{(i)}=(a_{12}^{(i)})^2-a_{11}^{(i)}a_{22}^{(i)}\\
&A_{kj}^{(i)}=a_{1k}^{(i)}A_{1j}^{(i)}+a_{2k}^{(i)}A_{2j}^{(i)}+a_{6k}^{(i)}A_{6j}^{(i)}+a_{kj}^{(i)},\quad A_{kj}^{(i)}\neq A_{jk}^{(i)};\ j,k=3,4,5\\
&D_{kj}^{(i)}=\zeta A_{kj}^{(i)}+A_{kj}^{(2)},\quad B_{12}^{(i)}=a_{12}^{(i)}a_{66}^{(i)}-a_{16}^{(i)}a_{26}^{(i)}\\
&B_{16}^{(i)}=(a_{16}^{(i)})^2-a_{11}^{(i)}a_{66}^{(i)},\quad B_{61}^{(i)}=a_{11}^{(i)}a_{26}^{(i)}-a_{12}^{(i)}a_{16}^{(i)}\quad(1,2),\quad B_{j6}^{(i)}\neq B_{6j}^{(i)}\\
&\Delta^{(i)}=a_{11}^{(i)}a_{22}^{(i)}a_{66}^{(i)}+2a_{12}^{(i)}a_{26}^{(i)}a_{16}^{(i)}-a_{11}^{(i)}(a_{26}^{(i)})^2-a_{22}^{(i)}(a_{16}^{(i)})^2-a_{66}^{(i)}(a_{12}^{(i)})^2
\end{aligned} \tag{1.12}$$

The quantities $Q_*^{(i),s}(\xi,\eta,\zeta)$ are known functions if the desired functions are known for the approximations $0,1,\ldots,(s-1)$. The following recursion formulas are obtained to evaluate them:

$$\sigma_{\alpha\gamma*}^{(i),s}=-\int_0^\zeta\left[\frac{1}{A}\frac{\partial\sigma_{\alpha\alpha}^{(i),s-1}}{\partial\xi}+\frac{1}{B}\frac{\partial\sigma_{\alpha\beta}^{(i),s-1}}{\partial\eta}+a\left(\sigma_{\alpha\alpha}^{(i),s-1}-\sigma_{\beta\beta}^{(i),s-1}\right)k_\beta+2ak_\alpha\sigma_{\alpha\beta}^{(i),s-1}+F_{\alpha s}^{(i)}\right]d\zeta\quad(\alpha,\beta;\xi,\eta) \tag{1.13}$$

$$\sigma_{\gamma\gamma*}^{(i),s} = -\int_0^\zeta \left[\frac{1}{A}\frac{\partial\sigma_{\alpha\gamma}^{(i),s-1}}{\partial\xi} + \frac{1}{B}\frac{\partial\sigma_{\beta\gamma}^{(i),s-1}}{\partial\eta} + ak_\beta\sigma_{\alpha\gamma}^{(i),s-1} + ak_\alpha\sigma_{\beta\gamma}^{(i),s-1} + F_{\gamma s}^{(i)}\right] d\zeta$$

$$\sigma_{\alpha\alpha*}^{(i),s} = -[B_{26}^{(i)}R_1^{(i),s} + B_{12}^{(i)}R_2^{(i),s} + B_{62}^{(i)}R_3^{(i),s}]/\Delta^{(i)} \quad (\alpha, \beta; 26, 12; 12, 16; 62, 61)$$

$$\sigma_{\alpha\beta*}^{(i),s} = -[B_{63}^{(i)}R_1^{(i),s} + B_{61}^{(i)}R_2^{(i),s} + A_{12}^{(i)}R_3^{(i),s}]/\Delta^{(i)}$$

$$u_*^{(i),s} = \int_0^\zeta [a_{15}^{(i)}\sigma_{\alpha\alpha*}^{(i),s} + a_{25}^{(i)}\sigma_{\beta\beta*}^{(i),s} + a_{35}^{(i)}\sigma_{\gamma\gamma*}^{(i),s} + a_{45}^{(i)}\sigma_{\beta\gamma*}^{(i),s} + a_{55}^{(i)}\sigma_{\alpha\gamma*}^{(i),s} +$$

$$a_{56}^{(i)}\sigma_{\alpha\beta*}^{(i),s} - \frac{1}{A}\frac{\partial w^{(i),s-1}}{\partial\xi} + \alpha_{13}^{(i)}\theta_s^{(i)}\Big] d\zeta \quad (u, v, 5, 4; \xi, \eta; 13, 23)$$

$$w_*^{(i),s} = \int_0^\zeta [a_{13}^{(i)}\sigma_{\alpha\alpha*}^{(i),s} + a_{23}^{(i)}\sigma_{\beta\beta*}^{(i),s} + a_{33}^{(i)}\sigma_{\gamma\gamma*}^{(i),s} + a_{34}^{(i)}\sigma_{\beta\gamma*}^{(i),s} +$$

$$a_{35}^{(i)}\sigma_{\alpha\gamma*}^{(i),s} + a_{36}^{(i)}\sigma_{\alpha\beta*}^{(i),s} + \alpha_{33}^{(i)}\theta_s^{(i)}]\, d\zeta$$

$$R_1^{(i),s} = \frac{1}{A}\frac{\partial u^{(i),s-1}}{\partial\xi} + ak_\alpha v^{(i),s-1} - a_{13}^{(i)}\sigma_{\gamma\gamma*}^{(i),s} - a_{14}^{(i)}\sigma_{\beta\gamma*}^{(i),s)} -$$

$$a_{15}^{(i)}\sigma_{\alpha\gamma*}^{(i),s} - \alpha_{11}^{(i)}\theta_s^{(i)} \quad (1, 2; \xi, \eta; u, v)$$

$$R_3^{(i),s} = \frac{1}{B}\frac{\partial u^{(i),s-1}}{\partial\eta} + \frac{1}{A}\frac{\partial v^{(i),s-1}}{\partial\xi} - a(k_\alpha u^{(i),s-1} + k_\beta v^{(i),s-1}) -$$

$$a_{36}^{(i)}\sigma_{\gamma\gamma*}^{(i),s} - a_{46}^{(i)}\sigma_{\beta\gamma*}^{(i),s} - a_{56}^{(i)}\sigma_{\alpha\beta*}^{(i),s} - \alpha_{12}^{(i)}\theta_s^{(i)}$$

It was assumed that the contribution of the volume forces and temperature effects is commensurate with the contribution of the surface forces and displacements when obtaining the relationships presented above. This will occur under the condition of validity of the asymptotic form (1.9), i.e., the volume forces should have sufficiently high intensity, otherwise the corresponding components will appear in the equations for the subsequent approximations. The large parameter ε^{-1} is inserted in the boundary conditions (1.3)-(1.5) for the same reason, to conserve the commensurability of the stresses caused by given displacements and surface forces.

The solution (1.7), (1.11) contains the still unknown functions $\sigma_{\gamma\gamma0}^{(s)}(\xi, \eta)$, $\sigma_{\beta\gamma0}^{(s)}(\xi, \eta)$, $\sigma_{\alpha\gamma0}^{(s)}(\xi, \eta)$, which are determined from conditions for $\gamma = h_1$ $(\zeta = h_1/h_2)$. By satisfying conditions (1.2), for instance, we obtain

$$\sigma_{\alpha\gamma0}^{(s)} = (-c_{344}V_\alpha^{(s)} + c_{354}V_\beta^{(s)} + c_{453}V_\gamma^{(s)})\Delta_1^{-1} \quad (\alpha, \beta; 4, 5) \tag{1.14}$$

$$\sigma_{\gamma\gamma0}^{(s)} = (c_{435}V_\alpha^{(s)} + c_{534}V_\beta^{(s)} - c_{455}V_\gamma^{(s)})\Delta_1^{-1}$$

$$\Delta_1 = D_{53}c_{435} + D_{54}c_{345} - D_{55}c_{344}, \quad D_{jk} = \zeta_0 A_{jk}^{(1)} + A_{jk}^{(2)}, \quad \zeta_0 = h_1/h_2$$

$$c_{jkl} = D_{jj}D_{kl} - D_{kj}D_{jl}, \quad j, k, l = 3, 4, 5$$

$$V_\alpha^{(s)} = u^{+(s)} - u^{-(s)} + u_*^{(2),s}(\zeta = -1) - u_*^{(1),s}(\zeta_0) \quad (\alpha, \beta, \gamma; u, v, w)$$

and

$$\sigma_{\alpha\gamma0}^{(s)} = \sigma_{\alpha\gamma}^{+(s)} - \sigma_{\alpha\gamma*}^{(1),s}(\zeta_0) \quad (\alpha, \beta), \quad \sigma_{\gamma\gamma0}^{(s)} = \sigma_{\gamma\gamma}^{+(s)} - \sigma_{\gamma\gamma*}^{(1),s}(\zeta_0) \tag{1.15}$$

corresponds to conditions (1.3), where $\sigma_{j\gamma}^{+(0)} = \sigma_{j\gamma}^{+}$ $(j = \alpha, \beta, \gamma)$, $\sigma_{\alpha\gamma}^{+(s)} = \sigma_{\beta\gamma}^{+(s)} = \sigma_{\gamma\gamma}^{+(s)} = 0$ for $s > 0$.

In the case of boundary conditions (1.4), we obtain

$$\sigma_{\alpha\gamma0}^{(s)} = \sigma_{\alpha\gamma}^{+(s)} - \sigma_{\alpha\gamma*}^{(1),s}(\zeta_0) \quad (\alpha, \beta) \tag{1.16}$$

$$\sigma_{\gamma\gamma0}^{(s)} = \frac{h_2}{h_1A_{33}^{(1)} + h_2A_{33}^{(2)}}[w^{+(s)} - w^{-(s)} + w_*^{(2),s}(\zeta = -1) - w_*^{(1),s}(\zeta_0) -$$

$$D_{34}\sigma_{\beta\gamma0}^{(s)} - D_{35}\sigma_{\alpha\gamma0}^{(s)}]$$

and for conditions (1.5) we have

$$\sigma_{\alpha\gamma0}^{(s)} = (D_{44}U^{(s)} - D_{54}V^{(s)})/(D_{44}D_{55} - D_{45}D_{54}) \tag{1.17}$$

$$\sigma_{\beta\gamma0}^{(s)} = (D_{55}V^{(s)} - D_{45}U^{(s)})/(D_{44}D_{55} - D_{45}D_{54}), \quad \sigma_{\gamma\gamma0}^{(s)} = \sigma_{\gamma\gamma}^{+(s)} - \sigma_{\gamma\gamma*}^{(1),s}(\zeta_0)$$

$$U^{(s)} = u^{+(s)} - u^{-(s)} + u_*^{(2),s}(\zeta = -1) - u_*^{(1),s}(\zeta_0) - D_{53}\sigma_{\gamma\gamma0}^{(s)} \quad (u, v)$$

Therefore, the solution of the internal problem is completely defined. As follows from (1.9), (1.11), (1.14)-(1.17), it contains no additional arbitrary constants. This class of problems is thus quite different from problems of the classical theory of plates and shells (the solution of the internal problem corresponding to the classical theory /1, 8/ contains constants that should be determined from conditions on the lateral surface). Since the solution of the internal problem contains no greater arbitrariness, and the system of boundary functions possesses the necessary completeness /9, 10/, then the conditions on the lateral surface in the problems considered here will generate only solutions of boundary-layer type, i.e., the boundary layer eliminates the residual on the lateral surface.

2. An exact solution of the internal problem can be obtained in a number of cases. For instance, when the anisotropy is rectilinear and the surface and volume forces of the temperature vary polynomially, the appropriate iteration process is truncated and a closed solution is obtained.

Let us examine some special cases when the plane $\gamma = -h_2$ is rigidly clamped: $u_\alpha(-h_2) = u_\beta(-h_2) = u_\gamma(-h_2) = 0$ and the following conditions are satisfied simultaneously.

1°. There are no volume forces and temperature changes, while a load of constant intensity acts on the face plane $\gamma = h_1$

$$\sigma_{\alpha\gamma}(h_1) = \sigma_{\alpha\gamma}^+, \quad \sigma_{\beta\gamma}(h_1) = \sigma_{\beta\gamma}^+, \quad \sigma_{\gamma\gamma}(h_1) = \sigma_{\gamma\gamma}^+; \quad \sigma_{j\gamma}^+ = \text{const.}$$

It follows from (1.9), (1.11), (1.15)

$$\begin{aligned} &\sigma_{\alpha\alpha}^{(i)} = A_{13}^{(i)}\sigma_{\gamma\gamma}^+ + A_{14}^{(i)}\sigma_{\beta\gamma}^+ + A_{15}^{(i)}\sigma_{\alpha\gamma}^+, \quad \sigma_{\beta\beta}^{(i)} = A_{23}^{(i)}\sigma_{\gamma\gamma}^+ + A_{24}^{(i)}\sigma_{\beta\gamma}^+ + A_{25}^{(i)}\sigma_{\alpha\gamma}^+ \\ &\sigma_{\alpha\beta}^{(i)} = A_{63}^{(i)}\sigma_{\gamma\gamma}^+ + A_{64}^{(i)}\sigma_{\beta\gamma}^+ + A_{65}^{(i)}\sigma_{\alpha\gamma}^+, \quad \sigma_{\alpha\gamma}^{(i)} = \sigma_{\alpha\gamma}^+, \quad \sigma_{\beta\gamma}^{(i)} = \sigma_{\beta\gamma}^+, \quad \sigma_{\gamma\gamma}^{(i)} = \sigma_{\gamma\gamma}^+ \\ &u_\alpha^{(i)} = h_2(D_{53}^{(i)}\sigma_{\gamma\gamma}^+ + D_{54}^{(i)}\sigma_{\beta\gamma}^+ + D_{55}^{(i)}\sigma_{\alpha\gamma}^+), \quad u_\beta^{(i)} = h_2(D_{43}^{(i)}\sigma_{\gamma\gamma}^+ + D_{44}^{(i)}\sigma_{\beta\gamma}^+ + D_{45}^{(i)}\sigma_{\alpha\gamma}^+) \\ &u_\gamma^{(i)} = h_2(D_{33}^{(i)}\sigma_{\gamma\gamma}^+ + D_{34}^{(i)}\sigma_{\beta\gamma}^+ + D_{35}^{(i)}\sigma_{\alpha\gamma}^+), \quad D_{kj}^{(i)} = \zeta A_{kj}^{(i)} + A_{kj}^{(2)} \\ &\zeta = \gamma/h_2, \quad i = 1, 2 \end{aligned} \tag{2.1}$$

2°. A normal load, linearly dependent on the coordinates α, β

$$\sigma_{\alpha\gamma}(h_1) = \sigma_{\beta\gamma}(h_1) = 0, \quad \sigma_{\gamma\gamma}(h_1) = b_1\alpha + b_2\beta$$

acts on an orthotropic plate. The first approximations of the iteration process are different from zero, and the following solution corresponds to them

$$\begin{aligned} &\sigma_{\alpha\alpha}^{(i)} = A_{13}^{(i)}(b_1\alpha + b_2\beta), \quad \sigma_{\beta\beta}^{(i)} = A_{23}^{(i)}(b_1\alpha + b_2\beta), \quad \sigma_{\alpha\beta}^{(i)} = 0 \\ &\sigma_{\gamma\gamma}^{(i)} = b_1\alpha + b_2\beta, \quad \sigma_{\alpha\gamma}^{(i)} = b_1(h_1A_{13}^{(1)} - \gamma A_{13}^{(i)}), \quad \sigma_{\beta\gamma}^{(i)} = b_2(h_1A_{23}^{(1)} - \gamma A_{23}^{(i)}) \\ &u_\alpha^{(i)} = b_1h_1A_{13}^{(1)}(h_2A_{55}^{(2)} + \gamma A_{55}^{(i)}) + \frac{1}{2}b_1(h_2^2A_{55}^{(2)}A_{13}^{(2)} - \gamma^2A_{55}^{(i)}A_{13}^{(i)}) - \\ &\frac{1}{2}b_1(h_2^2A_{33}^{(2)} + 2\gamma h_2A_{33}^{(2)} + \gamma^2A_{33}^{(i)}) \quad (\alpha, \beta; b_1, b_2; 5, 4; 13, 23) \\ &u_\gamma^{(i)} = (h_2A_{33}^{(2)} + \gamma A_{33}^{(i)})(b_1\alpha + b_2\beta), \quad i = 1, 2 \end{aligned} \tag{2.2}$$

3°. A constant normal displacement

$$\sigma_{\alpha\gamma}(h_1) = \sigma_{\beta\gamma}(h_1) = 0, \quad u_\gamma(h_1) = w^+ = \text{const}$$

is communicated to the surface $\gamma = h_1$.

The iteration is cut off in the initial approximation, which yields

$$\begin{aligned} &\sigma_{\alpha\alpha}^{(i)} = A_{13}^{(i)}a_1w^+, \quad \sigma_{\beta\beta}^{(i)} = A_{23}^{(i)}a_1w^+, \quad \sigma_{\alpha\beta}^{(i)} = A_{63}^{(i)}a_1w^+ \\ &\sigma_{\alpha\gamma}^{(i)} = 0, \quad \sigma_{\beta\gamma}^{(i)} = 0, \quad \sigma_{\gamma\gamma}^{(i)} = a_1w^+; \quad a_1 = (h_1A_{33}^{(1)} + h_2A_{33}^{(2)})^{-1} \\ &u_\alpha^{(i)} = (\gamma A_{53}^{(i)} + h_2A_{53}^{(2)})a_1w^+ \quad (\alpha, \beta; 5, 4), \quad u_\gamma^{(i)} = (\gamma A_{33}^{(i)} + h_2A_{33}^{(2)})a_1w^+ \end{aligned} \tag{2.3}$$

4°. We will calculate the temperature stresses in a double-layer orthotropic plate when the temperature change in the first layer is $\theta^{(1)}$, and in the second is $\theta^{(2)}$, where $\theta^{(1)} \approx \text{const}$ and $\theta^{(2)} \approx \text{const}$, there are no volume and surface forces $F_\alpha = F_\beta = F_\gamma = 0$; $\sigma_{\alpha\gamma} = \sigma_{\beta\gamma} = \sigma_{\gamma\gamma} = 0$ for $\gamma = h_1$, and $u_\alpha = u_\beta = u_\gamma = 0$ for $\gamma = -h_2$. Using (1.7), (1.9), (1.11)-(1.13) and (1.15) we obtain

$$\begin{aligned} &\sigma_{\alpha\alpha}^{(i)} = B_1^{(i)}\theta^{(i)}, \quad \sigma_{\beta\beta}^{(i)} = B_2^{(i)}\theta^{(i)}, \quad \sigma_{\alpha\beta}^{(i)} = -(\alpha_{12}^{(i)}\theta^{(i)})/a_{66}^{(i)} \\ &\sigma_{\alpha\gamma}^{(i)} = \sigma_{\beta\gamma}^{(i)} = \sigma_{\gamma\gamma}^{(i)} = 0, \quad i = 1, 2 \\ &u_\alpha^{(i)} = \alpha_{13}^{(2)}\theta^{(2)}h_2 + \alpha_{13}^{(i)}\theta^{(i)}\gamma, \quad (\alpha, \beta; 13, 23), \quad u_\gamma^{(i)} = B_3^{(2)}\theta^{(2)}h_2 + B_3^{(i)}\theta^{(i)}\gamma \\ &B_1^{(i)} = (\alpha_{12}^{(i)}\alpha_{22}^{(i)} - a_{22}^{(i)}\alpha_{11}^{(i)})/[a_{11}^{(i)}a_{22}^{(i)} - (a_{12}^{(i)})^2], \quad (B_1, B_2; 11, 22) \\ &B_3^{(i)} = a_{13}^{(i)}B_1^{(i)} + a_{23}^{(i)}B_2^{(i)} + \alpha_{33}^{(i)} \end{aligned} \tag{2.4}$$

If the temperature varies linearly over the layer thicknesses, $\theta^{(i)} = b_i\gamma + d$, there results from these same formulas

$$\begin{aligned} &\sigma_{\alpha\alpha}^{(i)} = B_1^{(i)}(b_1\gamma + d), \quad \sigma_{\beta\beta}^{(i)} = B_2^{(i)}(b_i\gamma + d) \\ &\sigma_{\alpha\beta}^{(i)} = -\alpha_{12}^{(i)}(a_{66}^{(i)})^{-1}(b_i\gamma + d), \quad \sigma_{\alpha\gamma}^{(i)} = \sigma_{\beta\gamma}^{(i)} = \sigma_{\gamma\gamma}^{(i)} = 0 \\ &u_\alpha^{(i)} = \alpha_{13}^{(2)}h_2\left(d - \frac{1}{2}b_2h_2\right) + \alpha_{13}^{(i)}\left(\frac{1}{2}b_i\gamma^2 + \gamma d\right) \quad (\alpha, \beta; 13, 23) \\ &u_\gamma^{(i)} = B_3^{(2)}h_2\left(d - \frac{1}{2}b_2h_2\right) + B_3^{(i)}\left(\frac{1}{2}b_1\gamma^2 + \gamma d\right) \end{aligned} \tag{2.5}$$

5°. Let the facial surface of the plate be rigidly clamped: $u_\alpha = u_\beta = u_\gamma = 0$ for $\gamma = h_1$; $-h_2$, there are no volume and surface forces, and the temperature varies in an arbitrary manner over the thickness (Fig.2). Taking account of (1.9), (1.11)-(1.13), we obtain for the expansion coefficients (1.7)

$$\sigma_{\alpha\gamma}^{(i),s} = h_2 (h_1 A_{55}^{(1)} + h_2 A_{55}^{(2)})^{-1} \left(\alpha_{13}^{(2)} \int_0^{-1} \theta_s^{(2)} \, d\zeta - \alpha_{13}^{(1)} \int_0^{h_1/h_2} \theta_s^{(1)} \, d\zeta \right) \tag{2.6}$$

$$u^{(i),s} = (\zeta A_{55}^{(i)} + A_{55}^{(22)}) \sigma_{\alpha\gamma}^{(i),s} - \alpha_{13}^{(2)} \int_0^{-1} \theta_s^{(2)} \, d\zeta + \alpha_{13}^{(i)} \int_0^{\zeta} \theta_s^{(i)} \, d\zeta$$

$$(\alpha,\ \beta;\ u,\ v;\ 13,\ 23;\ 5,\ 4)$$

$$\sigma_{\gamma\gamma}^{(i),s} = h_2 (h_1 A_{33}^{(1)} + h_2 A_{33}^{(2)})^{-1} \left(B_3^{(2)} \int_0^{-1} \theta_s^{(2)} \, d\zeta - B_3^{(1)} \int_0^{h_1/h_2} \theta_s^{(1)} \, d\zeta \right)$$

$$\sigma_{\alpha\alpha}^{(i),s} = A_{13}^{(i)} \sigma_{\gamma\gamma}^{(i),s} + B_1^{(i)} \theta_s^{(i)} \quad (\alpha, \beta; 1, 2), \quad \sigma_{\alpha\beta}^{(i),s} = -\alpha_{12}^{(i)} (a_{66}^{(i)})^{-1} \theta_s^{(i)}$$

$$w^{(i),s} = (\zeta A_{33}^{(i)} + A_{33}^{(2)}) \sigma_{\gamma\gamma}^{(i),s} - B_3^{(2)} \int_0^{-1} \theta_s^{(2)} \, d\zeta + B_3^{(i)} \int_0^{\zeta} \theta_s^{(i)} \, d\zeta$$

In particular, when $\theta^{(i)} \approx \text{const}$, a solution of the problem is

$$\sigma_{\alpha\alpha}^{(i)} = -A_{13}^{(i)} (B_3^{(1)} \theta^{(1)} h_1 + B_3^{(2)} \theta^{(2)} h_2)(h_1 A_{33}^{(1)} + h_2 A_{33}^{(2)})^{-1} + B_1^{(i)} \theta^{(i)} \tag{2.7}$$

$$\sigma_{\alpha\gamma}^{(i)} = -(\alpha_{13}^{(1)} \theta^{(1)} h_1 + \alpha_{13}^{(2)} \theta^{(2)} h_2)(h_1 A_{55}^{(1)} + h_2 A_{55}^{(2)})^{-1}$$

$$u_\alpha^{(i)} = -(\gamma A_{55}^{(i)} + h_2 A_{55}^{(2)})(\alpha_{13}^{(1)} \theta^{(1)} h_1 + \alpha_{13}^{(2)} \theta^{(2)} h_2)(h_1 A_{55}^{(1)} + h_2 A_{55}^{(2)})^{-1} + \alpha_{13}^{(2)} \theta^{(2)} h_2 + \alpha_{13}^{(i)} \theta^{(i)} \gamma \quad (\alpha, \beta; 13, 23; B_1, B_2; 5, 4)$$

$$\sigma_{\alpha\beta}^{(i)} = -\alpha_{12}^{(i)} (a_{66}^{(i)})^{-1} \theta^{(i)}, \quad \sigma_{\gamma\gamma}^{(i)} = -(B_3^{(1)} \theta^{(1)} h_1 + B_3^{(2)} \theta^{(2)} h_2)(h_1 A_{33}^{(1)} + h_2 A_{33}^{(2)})^{-1}$$

$$u_\gamma^{(i)} = -(\gamma A_{33}^{(i)} + h_2 A_{33}^{(2)})(B_3^{(1)} \theta^{(1)} h_1 + B_3^{(2)} \theta^{(2)} h_2)(h_1 A_{33}^{(1)} + h_2 A_{33}^{(2)})^{-1} + B_3^{(2)} \theta^{(2)} h_2 + B_3^{(i)} \theta^{(i)} \gamma$$

It is possible to write down solutions of problems corresponding to load and temperature changes using high-degree polynomials. For complex loads they can be approximated by a polynomial, then the appropriate exact solution can be written down.

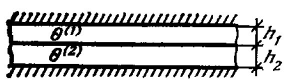

Fig.2

3. The asymptotic method can be used to compute elastic bases and foundations. A number of models of an elastic base exists /2, 4, 11/. These models can be classified as follows: 1) the Winkler-Zimmermann-Fusse model, or the model of the bed coefficient, 2) the model of a base with two elastic characteristics, or the model of a single-layer base (V.Z. Vlasov, P.L. Pasternak, M.M. Filonenko-Borodich), 3) the model of an elastic half-space (half-plane) with constant or variable elastic modulus over the depth, 4) the model of a compressible layer. The elastic modulus is considered constant or variable over the layer thickness (V.Z. Vlasov-N.N., Leont'ev, G.K. Klein, K.E. Egorov, et al.).

Compared with the Fusse-Winkler-Zimmermann model of an elastic base, the model of a compressible layer has the advantage that it enables the state of stress and strain of the basis itself to be assessed also. In the general case, the corresponding three-dimensional problem of thermoelasticity must be solved by this model in the general case. The model is described mathematically by the problem (1.1), (1.3). Having the solution of the problem, a judgment can be made on the validity of the kinematic hypotheses taken in /2/, and the validity of the Fusse-Winkler hypothesis can also be verified.

It follows from (2.1) that if the anisotropy is arbitrary, then even under a normal uniform load all the displacements are equally correct. It is natural that they remain the same even for tangential loads. If the base material is isotropic or orthotropic, then the normal displacement is the main one under the effect of a normal load, and the tangential displacements either equal zero as in the case of a uniform load, or are an order less as compared with a normal displacement under a non-uniform load. This can be seen by going over to dimensionless variables in (2.1), (2.2). It hence follows that the kinematic representations taken in /2/ hold for a uniform laod and are approximate, but sufficiently exact, for practical applications for non-uniform normal loads. These representations cannot be extended to the case of a foundation with arbitrary anisotropy.

From (2.1) we write down the values of the stresses and displacements for points of the contact surface (we denote them by the superscript c) corresponding to a normal load

$$\sigma_{\alpha\gamma}^c = \sigma_{\beta\gamma}^c = 0, \quad \sigma_{\gamma\gamma}^c = \sigma_{\gamma\gamma}^+ \tag{3.1}$$

$$\sigma_{\alpha\alpha}^{(i)}(\gamma=0)=A_{13}^{(i)}\sigma_{\gamma\gamma}^{+},\quad \sigma_{\beta\beta}^{(i)}(\gamma=0)=A_{23}^{(i)}\sigma_{\gamma\gamma}^{+},\quad \sigma_{\alpha\beta}^{(i)}(\gamma=0)=A_{63}^{(i)}\sigma_{\gamma\gamma}^{+},\ i=1,2$$

$$u_\alpha{}^c=A_{53}^{(2)}h_2\sigma_{\gamma\gamma}^{+},\quad u_\beta{}^c=A_{43}^{(2)}h_2\sigma_{\gamma\gamma}^{+},\quad u_\gamma{}^c=A_{33}^{(2)}h_2\sigma_{\gamma\gamma}^{+}$$

It follows from (3.1) that under the effect of a constant normal load the contact surface points always have tangential displacements proprotional to the load, where the proportionality factors are different for the different displacement components. It hence follows that when the anisotropy of the base is arbitrary, the Fusse-Winkler model is inapplicable. If orthotropy holds and the principal directions agree with the coordinates, then $A_{53}{}^{(2)}=A_{43}{}^{(2)}=0$ and there is a relationship $u_\gamma{}^c=A_{33}{}^{(2)}h_2\sigma_{\gamma\gamma}{}^c$ between the displacement of the point of contact and the reactive pressure, or

$$\sigma_{\gamma\gamma}^c=u_\gamma{}^c/1(A_{33}^{(2)}h_2) \tag{3.2}$$

i.e., the Winkler-Fusse hypothesis is satisfied exactly, and $k=1/(A_{33}{}^{(2)}h_2)$ is the bed coefficient. For the isotropic case, calculating the value of $A_{33}{}^{(2)}$ from (1.12), we obtain

$$k=\frac{(1-\nu_{(2)})\,E_{(2)}}{(1+\nu_{(2)})\,(1-2\nu_{(2)})\,h_2} \tag{3.3}$$

which agrees with the known bed coefficient proposed by N.M. Gersevanov (/4/, p.56). It follows from (2.3) that this same bed coefficient is obtained when the facial surface $\gamma=h_1$ receives a normal displacement, in the case of a temperature effect it follows from (2.4)-(2.7) that the above-mentioned proportionality does not exist.

If the normal load is variable, then the tangential displacements of points of the contact surface become different from zero in both the orthotropic and isotropic cases, and additional components to the Winkler ones appear in the expression for the normal displacement. However, it can be seen (see (2.2)) that these components are of an order of magnitude less than the Winkler ones. Consequently, although the Winkler-Fusse hypothesis is not satified exactly mathematically because of the variability of the external load, it is acceptable within known limits.

We also note that when we speak of the applicability of any model, the above is valid for the internal problem, i.e., at a distance from the lateral surface equal to the boundary-layer damping zone (edge effects), If other stress or displacement values than those that result from the solution of the internal problem are given on the lateral surface of the plate, then a boundary layer occurs where there is no proportional dependence between the normal displacement and the reactive pressure. In the case of one compressible layer, the plane boundary layer is studied in /9/ and the question of its interaction with the solution of the internal problem is discussed. The fundamental relationships of the spatial boundary layer are obtained in /10/. It is shown that the boundary layer damps out exponentially, while the value of the exponent depends on the elastic properties of the material. The boundary layer is constructed by the same method for laminar plates. We say that probems of laminar plates and shells with an arbitrary number of layers can be solved by the same asymptotic method even when the elastic moduli are variable over the thickness of each layer.

Contact problems of anisotropic shells and plates can be solved by using the solution of the internal problem and the boundary layers of the mixed problems mentioned.

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STABILITY OF CIRCULAR PLATES FROM AGEING VISCOELASTIC MATERIAL*

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Stability conditions are obtained for circular plates of an inhomogeneously-ageing viscoelastic material for an arbitrary creep kernel and different methods of plate support. Stability in an infinite time interval corresponds to determination of the Lyapunov stability, and in a finite interval, Chetayev stability.

1. Formulation of the problem. Consider the axisymmetric deformation of a circular plate of constant thickness h and radius R. We introduce a cylindrical $r\varphi z$ coordinate system whose origin is at the centre of the plate middle plane in the undeformed state, while the z-axis is perpendicular to this plane. At a time $t=0$ an external load is applied to the plate. We denote the age of the plate material at the point r at the time of external load application by $\rho(r)$. The function $\rho(r)$ is piecewise-continuous and bounded.

The stress σ_{ij} and strain ε_{ij} tensor components $(i, j = r, \varphi, z)$ are connected by the relationships

$$e_{ij} = (1+\nu)(I+L)\,s_{ij}/E,\quad \varepsilon = (1-2\nu)\,\sigma/E \tag{1.1}$$

$$s_{ij} = E\,(1+\nu)^{-1}(I-N)\,e_{ij},\quad \sigma = E\,(1-2\nu)^{-1}\varepsilon$$

$$\sigma = (\sigma_{rr}+\sigma_{\varphi\varphi}+\sigma_{zz})/3,\qquad \varepsilon = (\varepsilon_{rr}+\varepsilon_{\varphi\varphi}+\varepsilon_{zz})/3$$

$$e_{ij} = \varepsilon_{ij} - e\delta_{ij},\qquad s_{ij} = \sigma_{ij} - \sigma\delta_{ij}$$

$$Ix = x(t),\qquad Lx = \int_0^t l(t+\rho, \tau+\rho)\,x(\tau)\,d\tau,$$

$$Nx = \int_0^t n(t+\rho, \tau+\rho)\,x(\tau)\,d\tau$$

Here E is the constant modulus of elastic instantaneous deformation, ν is the constant Poisson's ratio, δ_{ij} are Kronecker deltas, I is the unit operator, L is the creep operator, N is the relaxation operator, and $l(t,\tau)$ and $n(t,\tau)$ are the creep and relaxation kernels.

The external load applied to the plate consists of a transverse distributed load of intensity $q(r)$ and compressive forces of constant magnitude p.

Let $w(t, r)$ denote the plate deflection at the point r at the time t, w_0 the maximum allowable value of the deflection, and T_0 the first time the deflection reaches the value w_0.

Definition 1. A plate is called Lyapunov stable in an infinite time interval if for any $\varepsilon > 0$ there exists a $\delta(\varepsilon) > 0$ such that the estimate $|w(t, r)| < \varepsilon$ $(t \geqslant 0,\ r \in [0, R])$ follows from the inequality $|q(r)| < \delta$

Definition 2. A plate is called stable in an interval $[0, T]$ if $T < T_0$.

The aim of this paper is to obtain the conditions for the magnitudes of the compressive forces p for which the plate is stable according to Defintions 1 and 2.

2. Governing equations. Suppose an axisymmetric generalized plane state of stress exists in the plate. Then $\sigma_{r\varphi}=0$ and the quantities σ_{iz} $(i = r, \varphi, z)$ can be neglected. We consequently obtain from (1.1)

$$\sigma_{rr} = E\,(1-\nu^2)^{-1}\,[(1-\nu)(I-N)\,\varepsilon_{rr} + \nu\,(I-K)\,(\varepsilon_{rr}+\varepsilon_{\varphi\varphi})] \tag{2.1}$$

$$\sigma_{\varphi\varphi} = E\,(1-\nu^2)^{-1}\,[(1-\nu)(I-N)\,\varepsilon_{\varphi\varphi} + \nu\,(I-K)(\varepsilon_{rr}+\varepsilon_{\varphi\varphi})]$$

$$K = N\,\{I - (1+\nu)\ (1-2\nu)\ (3\nu-3\nu^2)^{-1}\ [I + (1+\nu)\times(3-3\nu)^{-1}\,L]^{-1}\}$$

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